

Exploring Interactions of Charged Dust Particles in Charge Clouds

Mr.M.Uppa Mahesh

*Assistant Professor, Department of H&S,
Malla Reddy College of Engineering for Women.,
Maisammaguda., Medchal., TS, India*

Abstract:

Two charged dust particles inside a cloud of charges are considered as Debye atoms forming a Debye molecule. Cassini coordinates are used for the numerical solution of the Poisson-Boltzmann equation for the charged cloud. The electric force acting on a dust particle by the other dust particle was determined by integrating the electrostatic pressure on the surface of the dust particle. It is shown that attractive forces appear when the following two conditions are satisfied. First, the average distance between dust particles should be approximately equal to two Debye radii. Second, attraction takes place when similar charges are concentrated predominantly on the dust particles. If the particles carry a small fraction of total charge of the same polarity, repulsion between the particles takes place at all distances. We apply our results to the experiments with eremomycin plasma and to the experiments with nuclear-pumped plasma. © Central European Science Journals. All rights reserved.

Keywords: dust particle, Debye atom, Debye molecule, Poisson-Boltzmann equation, Cassini coordinates, eremomycin plasma, nuclear pumped plasma PACS (2000): 52.27.

Introduction

The study of a plasma in which charged particles of micrometre size play a significant role (so-called dust plasma) is interesting from the fundamental and applied points of view [16]. Of special interest is the observation of collective effects caused by dust coupling. A number of experiments show that micron size particles can form spatial-ordered structures in eremomycin plasma [3] in gas-discharge plasma and in nuclear-pumped plasma [4]. The properties of strong-coupled plasma are often considered in the framework of the so-called one-component model (see, for example, the review by [10]). According to this model, one of the charged components is treated as homogeneous in space. Polarization effects are taken into account in the form of corrections, in some cases. Apparently, the physics of processes occurring in dust plasma

differs from the one-component model. A dust particle surrounded by a shell (or cloud) of charges (with masses much smaller than the mass of the dust particle) should be object of detailed consideration, first of all. A charged dust particle surrounded by a cloud of charges of the opposite sign is an analogue of an atom in gas kinetics. In general, the charged cloud of such a "dust atom" may not be in thermodynamic equilibrium. However, we shall consider here the situation in which the charges in a cloud are Boltzmann-distributed. It is natural to call such a dust atom a Debye atom [15] in contrast to a Thomas-Fermi atom, in which a charged cloud is a degenerate electronic gas. Similarly, we can introduce the concept of a Debye molecule [17] and a Debye crystal. The Boltzmann distribution and the Poisson equation (that is, the Poisson-Boltzmann equation) describe mathematically the properties of such Debye systems. It is natural to assume the presence of attractive forces caused by polarization of the charge shells of Debye atoms. However, reliable theoretical results demonstrating an attraction of Debye atoms do not yet exist.

The exact solution of the Poisson-Boltzmann equation shows that the repulsion always takes place for the charged planes both in an electron cloud and in a plasma [2], [20]. Numerical simulation of Debye atoms interaction [17] were not quite reliable, as were the results of analytical calculations [5], [11]. The problem of particle interactions in dusty plasma is similar to the problem of colloidal particle interactions in electrolytes. The very concept of a Debye radius for plasmas was borrowed from the theory of electrolytes. The physics of colloid particle interactions in electrolytes has been investigated for a long time (see, for example, [2]). Until now, however, the problem of attraction forces has not been solved, at least for the case in which the colloid particle radius is smaller than the Debye radius. Below, we attempt to reliably demonstrate the existence of polarization forces of attraction between Debye atoms and to determine the conditions under which attraction appears. This work differs essentially from other publications devoted to an analysis of charged dust particle interactions in plasmas and in electrolytes (see, for example, [1], [13] and [14]). First, in contrast to a

number of publications, we consider a situation in which the total charge of dust particles is not negligibly small compared with the total charge of the cloud particles of one sign. Moreover, we show here that the essential attraction takes place in an opposite limiting case, that is, when almost all the charge of one of sign is concentrated on dust particles, and the clouds consist of charges of only one (opposite) sign. (See [7-9] for preliminary results.) Second, based on Debye molecule properties, a Debye atom has denoted structure. The Debye atom has a core of a charged cloud close to the surface of a dust particle, when a dust particle has a high charge. In particular, the charge of a dust particle cannot, as a rule, be considered as an approximate delta function, even if its radius is much smaller

than the Debye radius. Third, we calculate directly the resulting force on the dust particle from another particle and the charged cloud. The dependence of the potential energy of interaction on dust particle separation is calculated by integration of this force. In our case, the Poisson-Boltzmann equation is solved in an infrequently used coordinate system based on Cassini ovals. It allows a highly accurate calculation of an electric field near a small particle surface and reliably obtains the force of a particle interaction. We apply our results to eremomycin plasma and to nuclear pumped plasma.

Formulation of the problem

For the sake of denizens, we shall consider eremomycin plasma, and speak about positively charged dust particles and the electronic cloud of a dust particle. However, basic results are also of use for dust plasma produced by the electrical discharge and for plasma ionized by an external source of hard radiation, when the particles are charged negatively, and the charged cloud consists mainly of positive ions. We discuss the nuclear pumped dust plasma below. So, let us consider the case in which an electronic gas surrounding the charged dust particles is formed by the emission of electrons from dust particles at succulently high temperature T . In addition, the dust particles are surrounded by a partially ionized gas. In order to find the spatial distribution of the potential ϕ , the electric field intensity E , and the charge density $\rho = e(N_i - N_e)$, we must solve the Poisson equation $\nabla^2 \phi = -\rho/\epsilon_0 = -4\pi e(N_i - N_e)$. The ions and electrons densities (N_i and N_e) appearing in this equation are determined by the Boltzmann distribution $N_i = N_{i0} \exp(-e\phi/T)$ and $N_e = N_{e0} \exp(e\phi/T)$, where N_{i0} and N_{e0} are the ion and electron densities at the points of zero potential. Thus, the Poisson-Boltzmann equation takes the form:

$$\Delta\phi = 4\pi e \left(N_{e0} \exp\left(\frac{e\phi}{T}\right) - N_{i0} \exp\left(-\frac{e\phi}{T}\right) \right). \quad (1)$$

Dimensionless variables:

We shall measure length in units of Debye radius $rd. = (T/4\pi e^2 N_{e0})^{1/2}$ corresponding to electron density in points of zero potential. We use the dimensionless potential', electric field intensity E , and electronic density n :

Here $\pm = N_{i0}/N_{e0}$ is the parameter describing the additional ionization of gas. Since the plasma is quasi-neutral, one has $0 < \mu \pm < 1$. For further estimations, we shall be guided by the experiments of [3], in which $N_{e0} = 2.5 \times 10^{10} \text{cm}^{-3}$ and $T = 0.146 \text{eV} = 1700 \text{K}$. For characteristic values we have $rd. = 18 \text{microns}$, $T/e = 0.146 \text{V}$, and $T/\text{herd} = 80 \text{V/cm}$. The average radius of a dust particle was $rap = 0.4 \text{microns}$ ($r_0^2 rap = rd. = 2.23 \times 10^2$) and its charge was $Ze = 500e$. So, we have a electric field intensity on a particle surface $Ze/r_0 = 4.5 \times 10^4 \text{V/cm}$ ($E_0 = E(r_0) = 550$).

Boundary conditions:

We will use the term "Debye atom" for a single charged dust particle surrounded by a cloud of lighter charges in thermodynamic equilibrium; two or more dust particles will be referred to as a Debye molecule. Formally, the analyses of a Debye atom and a Debye molecule differ only in the geometry of the problem. While analysing a Debye atom, we can get by with the solution of the one-dimensional Poisson equation, assuming that the electron cloud is spherically symmetric. In an analysis of a diatomic Debye molecule, we can assume that the problem is symmetric about the x-axis connecting the nuclei (dust particles). Therefore, it is enough to consider the two-dimensional Eq. 3 in plane coordinates (x, y) . When analysing a Debye molecule, the problem is complicated considerably by the choice of boundary conditions. In a real physical problem, the charge Ze of a dust particle and its radius rap are species. Hence, one boundary condition is the electric field intensity on the surface of dust particles S :

$$E_n = -\nabla\phi|_S \quad (4)$$

Thus, the charge of a dust particle is determined by the expression

$$Z_p = \frac{-r_p^2}{4\pi e} \int_S \nabla\phi ds, \quad z_p = \frac{1}{4\pi} \int_S E ds. \quad (5)$$

The zero-electric field intensity on the Debye atom or molecular boundary follows from quasineutrality of the system of charges. The basic purpose of Debye

molecule consideration is to \bar{N}_d resulting dependence of the particles' interaction force on the distance d between particles. In this case, it is more convenient to use other boundary conditions instead of Eq. 4, that is, to set a constant potential on a surface of dust particles,

One can get the \bar{E}_0 intensity E_0 on a surface of a dust particle by solving the Poisson-Boltzmann equation. The calculations with different values of \bar{N}_d give the necessary value of E_0 and charge value z_p (Eq. 5). The resulting force of interaction of the dust particles is determined by integration of the electrostatic pressure on a surface of a dust particle. In one case the force is directed along an axis x , and its projection is determined by the expression

$$F = \frac{1}{8\pi} \int_S (\nabla\phi)^2 |S| ds_x, \quad f = \int_S E_0^2 ds_x \quad (8)$$

Here ds_x is a projection of surface element ds on an axis x ; the force F is connected to dimensionless force f by the expression $F = (T/8^{1/2} \epsilon e^2) \phi f$; the electric pressure is directed along the outward normal to the surface of dust particles.

Some properties of Debye atoms:

The properties of a Debye molecule in some aspects are denied by properties of the Debye atoms forming this molecule. Therefore, we shall consider some properties of Debye atoms before beginning calculation of the force of dust particles interaction. In the one-dimensional (that is, planar, cylindrically symmetric, or spherically symmetric) case, equation (3) and boundary conditions take the form

$$\frac{1}{r^k} \frac{d}{dr} \left(r^k \frac{d\phi}{dr} \right) = \exp(\phi) - \delta \cdot \exp(-\phi), \quad \phi|_{r=a_0} = 0, \quad E(r)|_{r=a_0} \equiv - \frac{d\phi}{dr} \Big|_{r=a_0} = 0. \quad (9)$$

Here $k = 0, 1,$ and 2 respectively for planar, cylindrically symmetric, and spherically symmetric cases; $r = 0$ corresponds to the beginning of a planar layer, centre of the cylinder, or centre of the sphere. One boundary condition sets the boundary of the Debye atom $r = a_0$, on which the \bar{E}_0 is equal to zero. The spherically symmetric case ($k = 2$) simulating a Debye atom and the \bar{E}_0 at case ($k = 0$), which allows us to study the potential variation near a dust particle surface, will be considered. In a spherically symmetric case, the convenient characteristic of a Debye atom is the dimensionless charge distribution (charge contained inside a sphere of radius r); it is denoted by the expression $z(r) = r^2 E(r)$.

Debye atom in a single-sign charge cloud

The case $\pm = 0$, in which the charged cloud consists of particles of one sign, corresponds, for example, to an eremomycin plasma [3] or a similar gas ionization process, in which the charges of one sign have completely concentrated on the dust particles [19]. Size a_0 we choose equal to half of the average distance between dust particles $a_0 = 40$ V.A. Rudenko, S.I. Yakovenko / Central European Journal of Physics 2(1) 2004 35{66 a p/r D² (N p/2r D), where N_p is the density of dust particles (see Figure 1). The size $a_p = 2N_p/2$ is 24% less than the Wigner-Zeitz radius: $r_{WZ} = (4^{1/4} N_p/3)^{1/3}$. Consider the most interesting situation, when a dust particle radius r_p is much less than the distance between dust particles $r_0 = r_p/r_0 = 1/2 a_0$. In experiments [3] $r_p = 0.4 \mu\text{m}$, $N_p = 5 \times 10^7 \text{ cm}^{-3}$, and $a_p = 13.6 \mu\text{m}$; thus, $a_p/r_p = a_0/r_0 = 34$. The results of equation (3) for the spherically symmetric case ($k = 2$) show that for the smaller charge $z_p = Z_p e / 2/r DT < a_0/3$ of a small particle $r_0 = 1/2 a_0$; the charge, \bar{E}_0 , and potential distributions are given by expressions [6]:

$$z(r) = (a_0^3/3) \cdot \left[1 - \left(\frac{r}{a_0} \right)^3 \right], \quad (10a)$$

$$E(r) = \frac{z(r)}{r^2}, \quad (10b)$$

$$\phi(r) = \left(\frac{a_0^3}{3} \right) \cdot \left[\frac{a_0}{r} - \frac{3}{2} + \left(\frac{r}{2a_0} \right)^2 \right]. \quad (10c)$$

The expressions 10 are still of use for the points far from a dust particle surface (at $r_0 + 3r_0/2 > r > r_0$) when the charge is high $z_p > a_0/3$. The variation from these expressions takes place close to a surface ($r_0 < r_0 + 3r_0/2 < a_0/3$), where a sharp fall of $z(r)$, $E(r)$, and $\phi(r)$ takes place (Figure 2). Otherwise, at the high charge of a dust particle, the Debye atom has some charged core close to a dust particle's surface. The charge of a dust particle together with the core is equal to $z_{\text{core}} = a_0/3$. The screening of this residual charge takes place at a large distance $r \gg a_0$. The high charge condition $z_p = Z_p e / 2/r DT > z_{\text{core}}$ can be written in the form

$$Z_p > Z_{\text{cor}} \equiv \frac{\pi N_d a_0}{6 N_p}$$

According to measurements of [3], the charge of dust particles was high:

$$Z_p = 500 > Z_{\text{cor}} = 262, \quad z_p = 0.273 > z_{\text{cor}} = 0.143.$$

However, calculations show (see Figure 2) that the dust particle charge in a thermal balance ($Z_p = 286$, $z_p = 0.156$) should be smaller than the value ($Z_p \approx 500$) measured by [3]. Hence, either the measurements of plasma parameters are not exact, or the charge of dust particles in experiments [3] is nonequilibrium (for details, see [18]).

Debye atom in plasma:

In the case $\pm \neq 0$, when the charged cloud consists of particles of both signs, the Debye atom radius, as before, is determined as the distance ($r = a_0$) at which the charge of a dust particle is completely compensated by plasma charges ($E(a_0) = 0$). As in the case $\pm = 0$, the Debye atom radius is equal to half of the average distance between dust particles $a_0 = \lambda_p / D^2 (N_p \approx 3 \lambda_p / 2r D)$. If $\pm = 1$, one isolated dust particle in an innate volume of V.A. Rudenko, S.I. Yakovenko / Central European Journal of Physics 2(1) 2004 35{66 41 plasma can be considered. If $\pm \neq 1$ the Debye atom radius tends to infinity: $a_0 \rightarrow \infty$. This is because the net charge of a particle z_0 can be completely compensated by a quasimetric plasma only at its innate sizes. If $\pm < 1$, the Debye atom radius is finite. Electronic and ionic dimensionless charges contained in a charged cloud are determined by expressions:

$$z_{0e} = \int_{r_0}^{a_0} \exp(\varphi(r)) r^2 dr, \quad z_{0i} = \delta \int_{r_0}^{a_0} \exp(-\varphi(r)) r^2 dr, \quad z_0 = z_{0e} - z_{0i}. \quad (11)$$

The quantity $\pm^2 z_0 / z_{0e}$ gives the relation of a free ion charge in Debye atom to an electron charge. Generally speaking, \pm^2 should be a function of the parameters \pm , a_0 and r_0 . However, when the main contribution to integration (11) is the area of a small potential $\varphi(r) \approx 1$, it is possible to put $\pm^2 \approx \pm$. Figure 3 illustrates the dependencies of z_{0e} , z_{0i} , and \pm^2 on \pm . In the results presented in Figure 3, the value of a_0 for different values of \pm was chosen as large as possible for the radius of a dust particle corresponding to the experiments of Forte et al. [3]: $r_0 = \lambda_p / D = 2.23 \times 10^{-22}$. This was carried out by "test ring": when the value of a_0 was chosen greater than that in Figure 3, the particle charge becomes innately large ($z(r_0) \rightarrow \infty$). The obtained dependencies $z(r)$ and $\varphi(r)$ (see Figure 4) were used to determine $z_0 = z(r_0)$ and $\varphi_0 = \varphi(r_0)$ at $r_0 = 0.1$ in the Debye molecule simulations presented below. The number of both positive and negative charge, z_{0i} and z_{0e} , in the cloud grows with \pm because of the increase of the Debye atom volume (see Figure 3). At the same time, the uncompensated charge $z_0 = z_{0e} - z_{0i}$ does not vary with changing \pm . At the considered parameters, we have $\pm^2 \approx \pm$. As well as in the case $\pm = 0$, at the given value r_0 , the size a_0 cannot be

innately large when a particle charge z_0 is innately high. The sharp fall of $z(r)$, $E(r)$, and $\varphi(r)$ as functions of r , caused by charge screening, takes place at distance $(r - r_0) \approx 1/E_0$ from a dust particle surface when value $E_0 = z_0 / r_0^2$ is high (see Figure 4). Thus, the size a_0 is limited by some value $a_{0max} \approx a_0(E_0 \approx 1)$. This limiting value increases logarithmically for $\pm \neq 1$:

$$a_{0max} \approx \frac{1}{2} \ln \left(\frac{1}{1 - \delta} \right) + \frac{1}{2}, \quad \text{when } 0.9 \leq \delta \leq 0.999.$$

Since a Debye atom has a core screening the charge of the dust particle, we cannot ascribe the unscreened value of the charge to the dust particle while considering the interaction of Debye atoms.

About the character of dust particles' interaction

polarized clouds the interaction force is expressed as

$$f(d) = \frac{z_{eff}(d) z_p}{d^2}.$$

Here d is the distance between dust particles and $z_{eff}(d) = E(d) d^2$ is a total charge that is taking place inside a sphere of radius d around of a dust particle. This is an uncompensated charge of a dust particle. Due to quasineutrality of the Debye atom, one has $z_{eff}(r) \approx 0$ at $r \approx r_0$. The charges of the same sign repel each other: $z_{eff}(d) z_p \approx 0$. The polarization of charged clouds is necessary for attractive forces. The number of negative charges should increase on the axis of a Debye molecule due to polarization if attractive forces take place.

The interaction of charged planes

The Poisson-Boltzmann equation (4) in a flat case ($k=0$) can be solved in quadratures. It obtains the interaction force of planes and obtains an accurate numerical solution of the Poisson - Boltzmann equation near the surface of a dust particle. This shows that the charged planes (both planes surrounded by a cloud of charges of the same sign, and planes located in the plasma) repulse each other [2], [20]. For an illustration we shall consider the case $\pm = 0$ to get simple analytical expressions. It is useful for an estimation of the necessary accuracy of calculations of a field and a potential near the surface of a dust particle. Consider the electrostatic pressure on the charged conducting plane, which is located between two conducting planes (left and right). The planes are under the potential φ_0 (see Figure 5). One of the planes can be removed to an infinite distance if necessary. The

integration of the Poisson-Boltzmann equation for a plane case gives [18], [20]:

$$\varphi(x) = \ln(E^2 + E_1^2), \quad E(x) = E_1 \cdot \tan \left[\frac{(a_0 - x)E_1}{2} \right].$$

The quantities $E_1 \equiv \exp(\varphi_1/2)$ and φ_1 are connected to a_0 by the expression:

$$a_0 = \left(\frac{2}{E_1} \right) \cdot \arctan \left(\frac{E_0}{E_1} \right).$$

Here x is the distance from the central plane, which for simplicity is treated as infinitely thin; φ_1 is the potential value in a point $x = a_0$, where the field intensity is equal to zero. Value a_0 is equal to half the distance between planes if the density of charges on the planes is equal. The potential at the left side and on the right side of the conducting plane is identical, $\varphi(-x) = \varphi(+x) = \varphi_0$. But a field intensity on a surface of the plane at the left side $E(-x) = 2E_0$ and at the right-side $E(+x) = 2E_0$ diverges because the distance from the central plane to the left plane $2a_0$ and to the right plane $2a_0$ diverges. Thus, an electrostatic pressure on a plane is:

The size a_0 is the monotonously falling function of E_1 . If, for example, distance to the left plane $2a_0$ is more than the distance up to the right plane $2a_0$; we have $E_0 > E_1$ and $p < 0$. Otherwise, the resulting pressure force is directed to the most removed plane. In particular, if we remove one of planes to an infinite distance, two planes will repulse. Thus, the attraction of dust particles can arise only in a geometry that is not flat. 3.2.3 Accuracy of the potential calculation near the surface in the numerical integration of the Poisson-Boltzmann equation, the value of the field intensity is determined in the grid points of a reference scheme. The value E_0 , determined approximately, corresponds to a field value some distance from a dust particle surface, of the order of a grid step. Let us estimate the error of calculated pressure. The relative difference of pressure determined at distances x and $\{x$ from a plane is given in flat geometry by the expression

$$\frac{\Delta p}{p} = \frac{|p - (E^2(-x) - E^2(x))|}{p} \quad (12)$$

As one can see in Figure 6, if the potential of a plane is not small ($\varphi \approx 1$), even on small distances $x \approx 0.01$, the value $\Delta p/p$ is in the approximate range of tens percent. At the same time, the difference of potentials at the left and on the right sides ($\varphi(-x) - \varphi(+x)$) is practically equal to zero. Otherwise, the very high accuracy of calculation of the potential derivative near a dust particle surface requires

numerical integration. Therefore, it demands a very small grid step near the surface. Distances between dust particles much exceeding their diameter are of the most interest. At the same time, the method used for the numerical integration of the Poisson-Boltzmann equation should provide the maximal accuracy in the area near the surface of dust particles for an exact calculation of force on a dust particle. It is difficult to achieve scientific accuracy in the calculation of force on small dust particles in the usual systems of coordinates.

The method of a two-centre problem solution

We used orthogonal coordinates constructed based on a known Cassini oval for a special case. The relationship between variables u and v , specifying a point on Cassini oval with the Cartesian coordinates in quadrant $x > 0, y > 0$, is determined by the following expressions:

$$x(u, v) = \frac{d}{2\sqrt{2}} \sqrt{\exp(2u) + 2\exp(u) \cdot \cos(v) + 1 + \exp(u) \cdot \cos(v) + 1}, \quad (13a)$$

$$y(u, v) = \frac{d}{2\sqrt{2}} \sqrt{\exp(2u) + 2\exp(u) \cdot \cos(v) + 1 - \exp(u) \cdot \cos(v) - 1}. \quad (13b)$$

The oval focus is located in point $(d/2, 0)$. Variable $1 > u > -1$ is some analogue of a radial variable. At $u > 0$ it represents an oval with a waist, and at $u > 0.65$ the oval has the ellipsoidal form. Variable $1/4 > v > 0$ is an analogue of a corner in polar coordinates. At $v = 0$ points lay on a beam $(d/2, 1)$ on the x -axis, at $v = 1/4$, the points come close to a corner formed by a line segment $(0, d/2)$ on the abscissa and beam $(0, 1)$ on the ordinate. The character of coordinate lines is illustrated in Figure 7. Use of coordinate (13) gives the following important advantages. First, the family of Cassini ovals qualitatively corresponds to an equipotential curve for two equally charged particles that are located in oval focuses. Second, the domain of the solution of equation (3) in these coordinates becomes rectangular. Third, the density of ovals is exponentially condensed to a surface of a dust particle. It makes an opportunity to use a uniform mesh even at the large distances between particles of small sizes.

On the method of numerical simulation

Without going into details, let us discuss the basic items of the numerical simulation method. The Cassini coordinates are especially convenient for use in a situation in which the radius of dust particles r_0 is much less than the Debye radius $r_D \approx 1$, and the radius of the Debye atom $r_0 \approx 1/2$.

is convenient to define the potential value φ_0 on small ovals close to circles. At the same time, the cloud of charges covering dust particles is described by an elliptical oval. It is convenient to set the field value to zero at this oval. The surface of a dust particle and the surface corresponding to the boundary of a Debye molecule are described in coordinate (13) by constants:

$$u_{\min} = \ln\left(\frac{4r_0}{d^2}(d+r_0)\right), \quad u_{\max} = \ln\left(\frac{4a_0}{d^2}(d+a_0)\right) \quad (14)$$

The boundary conditions (7) thus look like:

$$\varphi|_{u=u_{\min}} = \varphi_0, \quad \frac{\partial \varphi}{\partial u}|_{u=u_{\max}} = 0 \quad (15)$$

The Poisson-Boltzmann equation (3) with boundary conditions (14) and (15) was solved by a Gauss-Newton method of iterations with use of the software package MATLAB. Figure 8 shows plots of an equipotential surface in two coordinate systems. The three-dimensional coordinates formed by rotation of φ at coordinates (13) around the x -axes are used to calculate the charge (5) and the interaction forces (8) of dust particles. The interaction energy of dust particles was calculated using the formula

$$U(d) = - \int_{\infty}^d f(x) dx + \text{const.} \quad (16)$$

Results of calculations

The calculations were carried out for such parameters φ_0 , r_0 , and a_0 that correspond to an isolated Debye atom when $d \approx 4a_0$. For this purpose, the spherically symmetric problem (9) was solved and the potential φ_0 on a particle surface for given r_0 and a_0 was calculated. Then the two-centered problem for $d = 10a_0$ was solved using values φ_0 , r_0 , and a_0 . The results for the spherically-symmetric problem and for the two-centre problem coincided with high accuracy. Smaller values of d were used in the further series of calculations. In a series of calculations shown in Figure 9, we were guided by plasma parameters of experiments by [3], and have put $\varphi_0 = 0.755$. The calculations show that the area at large distances $d \approx 2a_0$ is most interesting. Therefore, we have taken the radius of a dust particle $r_0 = 0.1$ (several times greater than in the experiment). Accordingly, potential $\varphi_0 = 1.16$, taken from the one-centre problem solution for $r_0 = 0.1$, was smaller than the potential on a surface of a dust particle of small radius ($\varphi_0 = 6.5$ at $r_0 = r_p/r_D = 2.23 \cdot 10^{-2}$). Otherwise, the small conducting ball was replaced

by a conducting ball of greater size, with a charge partially compensated by charges of an electronic cloud. Such replacement is just because the electrons situated near the dust particle surface are weakly polarized (see below).

Interaction force dependence on dust particle separation

A series of calculations with the given values φ_0 , r_0 , and a_0 were carried out for different values of d . The dust particle charge z_0 is also a function of d in this case. Additional calculations were carried out with changed φ_0 or a_0 to make the dust particle charge z_0 not dependent on d . The calculations have shown that the repulsion takes place at small distances between particles $d \approx r_0$. It is not in accord with results of numerical calculations of [17], in which the dust particle attraction took place at $d \approx r_0$. Apparently, there was some error in the calculations of electric field near the surface of the dust particle. The resulting force is very sensitive to such error (see 3.2). Actually, the charged cloud is weakly polarized close to the surface of a dust particle, so the repulsion force prevails over the polarizing attraction force at small distances. Figure 9 shows that the dust particles' interaction force has zero value at equilibrium point $d = d_0 \approx 1.3$ under the conditions of [3]. The position of the equilibrium point $d = d_0$, in which a sign of interaction force changed, is less than the average distance between dust particles ($2a_0 = 1.5$). The value d_0 weakly depends on which quantity (φ_0 , a_0 , or z_0 , a_0) was kept constant in calculations at different d . The change a_0 (at constant $z_0 = 46$ V.A. Rudenko, S.I. Yakovenko / Central European Journal of Physics 2(1) 2004 35-66 and φ_0) nuances the value of d_0 some more. Apparently, it is better to make $z_0 = \text{const}$ by changing the dust particle potential $\varphi_0 = \varphi_0(d)$. It is impossible to consider a problem binary when $d \approx 4a_0$. The essential repulsion from other dust particles takes place if the distance between dust particles is large ($d > 2a_0$) (see a Figure 1). Therefore, we present the results of calculations only for $d < 4a_0$. One can estimate the electrostatic pressure compressing a dust particle gas as a function of an attraction force of dust particles $F(d)$ at average distance $2a_0$

Note, however, that the comparison of electrostatic pressure on dust particle gas with gas-kinetic electronic pressure does not allow one to make any essential conclusions. Electrons are not free; they are in an electrical field of dust particles. At the same time, it is possible to assume that the gas of Debye atoms in a mix with inert gas should show the tendency to compression under the conditions of the experiments. Such a situation was considered by [12]. Consideration of the impedance of Debye atoms' interaction on the gas-kinetic property of

dusty plasma is outside the framework of this paper.

Nuance of the Debye atom size:

The results of some series of calculations for various values a_0 are presented on Figure 10. The calculations have shown that the attraction of the dust particles takes place only at $a_0 \mu 1$. Already at $a_0 > 1.12$, the equilibrium point d_0 goes to large distance $d_0 > 4a_0$. It is possible to rewrite the condition $a_0 = \mu p/r D < 1$ for the dimensional quantity,

On the effect of a dust particle size:

The small charged ball is replaced above with a ball of greater size and accordingly with a partially compensated charge. There is a natural question whether such a replacement is adequate. Some series of calculations were done with deferent values of r_0 and coresponding values of ϕ_0 . The change in the results of the calculations is insignia if a dust particle radius is small in comparison with the radius of Debye atom a_0 . For example, in a case $a_0 = 0.755$ (see Figure 11) for $r_0 = 0.1 \mu 0.2$ (and for the choice of values of ϕ_0 corresponding to the given r_0), the divergences in the equilibrium point $d_0 = 1.28$ is less than 2%, which corresponds to the available accuracy of calculations. The elect of the dust particle size becomes Signiant for $r_0 > 0.3a_0$. For $r_0 > 0.4$, the polarization-induced attraction decreases to such an extent that the distance to the force sign-reversal point becomes larger than the mean distance between particles ($d_0 > 2a_0$). Therefore, it is possible to conclude that the electrons placed at distance r_0 ($0.3 \mu 1$) a_0 are involved in polarization. In this connection, it is dicot to hope for an analytical evaluation of attraction forces

A Debye molecule in a plasma ($\pm \phi = 0$):

As in the case of $\pm = 0$, the series of calculations were carried out to obtain the dependence of the interaction force of dust particles on distance d . The additional calculations were carried out with changed ϕ_0 or a_0 to make the dust particle charge z_0 independent of d . As in the case of $\pm = 0$, we chose the value of r_0 greater than the radius of the atomic core, thus simulating a dust particle by a conducting sphere of a larger size, with a charge partially compensated by the free charges from the shell of the Debye atom. Thus, the polarization of the core was disregarded. In the results shown in Figure 12, the size a_0 for deferent values of \pm corresponds to the extremely large charge of a dust particle with radius $r_0 p/r D = 2.23 \phi 10_2$. This was done by test \bar{r} -ring: when the value of a_0 was greater than that given in Table 7, the particle

charge obtained by solving Eq. (9) becomes innately large ($z_0 (r_0 p/r D) \mu 1$). The obtained dependencies $z_0 (r_0)$ and $\phi_0 (r_0)$ were used for determining $z_0 = z_0 (r_0)$ and $\phi_0 = \phi_0 (r_0)$, at $r_0 = 0.1$. We did not get an evident attraction of dust particles at $1 \pm \mu 1$ in the range of parameter $d < 2a_0$ that corresponds to binary interaction (Figures 12a, 12b). The attraction arises when an appreciable share of a positive charge of plasma is carried with dust particles (at $\pm < 0.7$, see Figures 12c, 12d). The maximum attraction force and the maximum depth of a potential well arises when $\pm = 0$. The decrease of an attraction force with growth of \pm has a simple explanation. As follows from the above calculations for $\pm = 0$, the attraction forces arise because electrons accumulate near the centre between dust particles and provide an attraction of positively charged dust particles to the centre of Debye molecule. This attraction force exceeds the repulsion force of dust particles because the Debye atom core screens the dust particle charge. At $1 \pm \mu 1$ the elect of a charge screening by the core is the same. However, the attraction force essentially weakens because not only electrons but also positive charges are accumulated near the centre of the Debye molecule. In case of a small value of a plasma charge $\pm \mu 1$, the potential well depth is rather great. It is about several values of gas temperature. However, the binary consideration is limited in size of the order of magnitude of a diameter of Debye atom $2a_0$ ($N_1 = 3 p > 2a_0 r$

On the analytical approaches:

The above conclusion concerning the absence of attraction for $\pm \mu 1$ contradicts the results of recent approximate analyses by [5], [11] (see Figure 12a). It follows from these analyses that the attraction of dust particles takes place at $\pm = 1$ and at interparticle separation $d > (31=2+1) /21=2 = 1.93$ if the linearized Poisson-Boltzmann equation is used. This result is surprising. In the linear approximation a potential in a point r is determined by the sum of the screened potentials of point charges located in points r

Inaccuracy of the results of [5] and [11] is apparently associated with the following circumstance. Gerasimov and Sinkevich [5] and Ivanov [11] sum the attraction force acting on an electronic cloud of the \bar{r} -rest dust particle from the second dust particle, and force (19) acting directly on a \bar{r} -rest dust particle. Such addition would be justice if the charged clouds of dust particles were rigidly connected with the dust particle through some other forces. However, there are no extraneous rigid forces in the problem under consideration. The presence of the attraction force of the electron shell of one charge to another charge only indicates that the given comigration of the charge shell is not in equilibrium. This force of

attraction must lead to polarization of the charge shell. Nevertheless, the polarization was disregarded by [5] and [11]. There are no grounds to add this polarizing force to the force acting directly on the dust particle. An analogous situation is the polarizing attraction of ordinary atoms. As is well known, for spherically symmetric atoms the polarizing interaction has no place in first-order perturbation theory. It arises only in second-order perturbation theory, when the polarization of an electronic shell of one atom by charges of other atom is taken into account. An ordinary atom differs from a Debye atom only by the fact that its electrons move according to quantum-mechanical and not classical laws. The nature of polarization-induced forces is the same for an ordinary and a Debye atom.

Dust particles in nuclear-pumped plasma

Experimental results:

Forte et al. [4] reported on the collective phenomena observed in dust plasma formed because of dense gas ionization by nuclear fission fragments. In one of these experiments, the plasma was excited by Cf252 fission fragments, and in the other by-products of the Ce141 β -decay. We will concentrate on the latter data. The dust was composed of CeO₂ particles with an average radius of $r_p = 0.5 \mu\text{m}$. The gravity force was compensated by applying an external electric field with a strength of 10 V/cm. The system featured large regions of particles levitating over several minutes, exhibiting a short-range order in the spatial structure. 50 V.A. Rudenko, S.I. Yakovenko / Central European Journal of Physics 2(1) 2004 35-66 Measurements performed using a digitized video image of the structure of these zones shows the density of particles within a 150- μm -thick flat layer was $10.5 \mu\text{m}^{-2}$. Accordingly, the volume density of dust particles was $N_p = 6 \times 10^4 \text{ cm}^{-3}$. The average charge of these particles, determined from the balance of gravitational and electrical forces, was $Z_p \approx 400$. The density of the charge of dust particles was $Z_p n_p = 2.4 \times 10^7 \text{ cm}^{-3}$. The ion density, determined by measuring the current between electrodes and using the known ion drift velocity, was $N_i = 108 \text{ cm}^{-3}$. The attraction of dust particles causes the collective phenomena in the nuclear-pumped plasma under consideration. As was stated above, the attraction of dust particles takes place if the charges of one sign are concentrated mainly on dust particles. Now we will check whether this condition is filled [19].

The charge of dust particles:

A negative charge on the dust particle surface may arise from a difference between average velocities of electrons and ions. This phenomenon is well

known in physical electronics. Assuming the Maxwell velocity distribution and equating the fluxes of ions to the particle surface $N_i v_{i1}$ to that of electrons $N_e v_{e2} \exp(-e\phi_p/kT_e)$, one obtains:

$$\phi_p = \left(\frac{T_e}{2e} \right) \cdot \ln \left(\frac{N_i m_i T}{N_e m_e T_e} \right).$$

Here, ϕ_p is the dust particle potential; $v_{i1} = (T_e/4\pi m_i)^{1/2}$ and $v_{e2} = (T_e/4\pi m_e)^{1/2}$ are the average projections of the velocities of ions and electrons onto the axis perpendicular to particle surface; and the T_e and T are the electron and gas temperatures. Using this potential value, we may formally determine the charge of the particle:

$$Z_p \approx \frac{r_p \phi_p}{e} \approx \left(\frac{r_p T_e}{2e^2} \right) \cdot \ln \left(\frac{N_i m_i T}{N_e m_e T_e} \right).$$

This estimate applies well to gas-discharge plasma, but may lead to considerable errors in the case of a plasma produced by a hard ionizing. Taking the temperature equal to the room temperature ($T_e = T = 300 \text{ K} = 0.026 \text{ eV}$), we obtain $Z_p \approx 100$. This estimate is about one-fourth of the value obtained from the experimental data ($Z_p \approx 400$). Apparently, the discrepancy is related to the fact that the secondary electron adheres to a dust particle before it is cooled in collisions with gas molecules.

Density of ions:

The charge-balance equation and the quasi-neutrality condition describe the number densities of ions and electrons in the dust plasma. In the case under consideration, these relationships can be written as follows:

$$\frac{dN_i}{dt} = G - \alpha_d N_i N_e - \alpha_L N_i N_p, \quad N_e = N_i - Z_p N_p$$

Here α_d is the dissociative recombination coefficient and α_L is the Langevin recombination coefficient; G is the ionization rate per unit volume. Under stationary conditions ($dN_i/dt = 0$), we may solve the above quadratic equation and present the ratio of the ion density N_i to the charge density $Z_p n_p$ in the following form:

$$\frac{N_i}{Z_p N_p} = \sqrt{\left(\frac{a-1}{2} \right)^2 + a} - \frac{a-1}{2}. \quad (20)$$

Here, $a = \alpha_L / (Z_p \alpha_d)$ is a parameter characterizing the ratio of the rates of the Langevin and dissociative recombination (for $a > 1$,

recombination on the dust particles dominates), and $g = G/(\alpha_{\text{Lapp}})$ is the reduced rate of ionization. Note an important circumstance: for an ionization rate satisfying the condition $g = 1$ or $G = \alpha_{\text{L}} Z p$ $\epsilon N p$, all the negative charge in the system is concentrated on the dust particles ($N_e = 0$, $N_I = Z p n p$) while the gas contains only positive ions. In the experiments under consideration, the radioactive source provided ionization at a rate corresponding to 10^9 -decays per second in a volume of 20 cm³. Assuming that every γ -decay event liberates an energy of $E_f = 138$ keV, we obtain the following estimate for the ionization rate per unit volume:

$$G \sim \left(\frac{10^9 \cdot s^{-1}}{20 \cdot \text{cm}^3} \right) \cdot \left(\frac{E_f}{E_{pr}} \right) \sim 2 \cdot 10^{11} \cdot s^{-1} \text{cm}^{-3}.$$

Here $E_{pr} = 36$ eV is the energy necessary for the ion pair production in air. The coincident of ion recombination on dust particles according to Langevin is $\alpha_{\text{L}} = 4/3 Z p e^{2b_i} \cdot 0.064 \epsilon \text{cm}^3 \text{ s}^{-1}$. This estimate is obtained for the ion mobility $b_i = 2/(\mu k_{\text{IA}})$, where $k_{\text{IA}} = (4/3) \epsilon 4 \epsilon 10_{16} \text{ cm}^2 \epsilon (8T / 4 \epsilon \text{mi})^{-1} = 2 \cdot 2.5 \epsilon 10_{11} \text{ cm}^2/\text{s}$ is the rate of collisions between ions and air molecules, considered as hard spheres with a cross section of $4 \epsilon 10_{16} \text{ cm}^2$. The ion density and the share of the dust particles charge $\pm = Z p n p = N_I$ can be estimated using (20). We use the dissociative recombination coincident equal to $\alpha_{\text{d}} \cdot 3 \epsilon 10_{7} \text{ cm}^3/\text{s}$ and take into account that recombination on the dust particles dominates: $\alpha = \alpha_{\text{L}} / (Z p n p) \cdot 530$. The estimated number density of ions $N_I \cdot 0.5 \epsilon 10^8 \text{ cm}^{-3}$ agrees with the experimental values. Moreover, expression (20) shows that, for the parameters under consideration, the negative charges are concentrated appreciably on dust particles, $\pm = Z p n p = N_I \cdot 0.5$. Thus, the attraction of dust particles can take place in these experiments.

Conclusion

Let us summarize the results of the above consideration. (1) A Debye atom consists of a charged dust particle and shell (cloud of charges). For the large charge of the dust particle, the high-density region (core) of the electron cloud screens considerably the large charge of the dust particle near its surface. In 52 V.A. Rudenko, S.I. Yakovenko / Central European Journal of Physics 2(1) 2004 35-66 this connection, while considering the interaction of Debye atoms, we cannot ascribe the unscreened value of charge to a dust particle. The dust particle charge screened by the core has a universal value determined by the distance between dust particles. The electron shell of the Debye atom screens it. (2) Attractive forces are associated with the polarization of charge shells of Debye atoms.

The force of attraction is formed by polarization of a large fraction of electrons of the charge shell. The polarization of the core is insignia. (3) Forces of attraction between dust particles emerge at a comparatively large distance, approximately equal to the mean separation between dust particles. In this case, the Debye radius must be approximately equal to half the mean distance between dust particles. (4) Attraction takes place if like charges are concentrated predominantly on dust particles. If dust particles carry a small fraction of the charge of some polarity, repulsion is observed at any distance. (5) The electrostatic forces of interaction between dust particles vanish when a certain relation between the electron density and the density of dust particles converges. In this case, the Debye "liquid" is in equilibrium. Since attractive forces appear at large distances, the problem of the formation of dust liquids and crystals can be solved correctly only if many-particle interactions are taken into account. However, we can draw the following two conclusions concerning the criteria for the emergence of collective phenomena based on the results presented by us here: (a) in the case of a thermionic plasma, the electron density must be such that the Debye radius is approximately equal to half the mean value between dust particles; (b) for a gas-discharge or a nuclear-excited plasma, the properties of the ionization source and the density of dust particles must be matched so that the main (usually negative) charge is carried by dust particles.

Acknowledgments

The authors are grateful to A.N. Tkachev for fruitful discussion of the results of the present work and also Yu. I. Setsuko for discussion of computational aspects of the problem.

References

- [1] W.R. Bowen and A.O. Sarif: \Long-range electrostatic attraction between like-charge spheres in a charged pore\, Nature (London), Vol. 393(18), (1998), pp. 663-665.
- [2] B. Derain and L. Landau: \Theory of Stability of Strongly Charged Lyophobic Sols and the Adhesion of Strongly Charged Particles in Solutions of Electrolytes\, Acta Physicochemical U.R.S.S., Vol. 14(6), (1941), pp. 633-662. V.A. Rudenko, S.I. Yakovenko / Central European Journal of Physics 2(1) 2004 35-66
- [3] V.E. Forte, A.G. Nemerov, O.F. Petrov, A.A. Samarian and A.V. Chernihiv \Hardly imperfect classical thermal plasma: experimental study of ordered structures of macroscopic particles\, HTC, Vol. 111(2), (1997), pp. 467-477 (in Russian).

- [4] V.E. Forte, V.I. Vladimirov, L.V. Depurative, V.I. Molotov, A.G. Nemerov, V.A. Rakove, V.M. Torchinsky and A.V. Hudak: \Ordered structures in nuclear pumped plasma\, Do lady Akademie Neuk, Vol. 336(2), (1999), pp. 184{187 (in Russian).
- [5] D.N. Gerasimov and O.A. Sinkevich: \Forming of ordered structures in thermal dusty plasma\, Teplizumab Vaisakhi Temperature, Vol. 37(6), (1999), pp. 853{857 (in Russian).
- [6] E.G. Gibson: \Ionization Phenomena in a Gas-Particle Plasma\, The Phys. of Fluids. Vol. 9(12), (1966), pp. 2389{2399.
- [7] V.A. Rudenko and S.I. Yakovenko: \The Interaction between Charged Dust Particles Calculated in Cassini Coordinates\, Technical Physics Letters, Vol. 28(5), (2002), pp. 422{426.
- [8] V.A. Rudenko and S.I. Yakovenko: \The Interaction between Charged Dust Particles in Plasma\, Technical Physics Letters, Vol. 28(11), (2002), pp. 919{922.
- [9] V.A. Rudenko and S.I. Yakovenko: \Interaction of Charged Dust Particles in Clouds of Thermodynamically Equilibrium Charges\, Journal of Experimental and Theoretical Physics, Vol. 95(5), (2002), pp. 864{877.
- [10] S. Ishimaru: \Strongly coupled plasmas: high-density classical plasmas and degenerate electron liquids\, Rev. Mod. Phys., Vol. 54, (1982), pp. 1017{1059.
- [11] A.S. Ivanov: \Polarization's Interaction and bound States of like charged Particles in Plasma\, Physics Letters A., Vol. 290, (2001), pp. 304{308.
- [12] S.A. Mayorov, A.N. Tkachev and S.I. Yakovenko: \Metastable state of Supercooled Plasma\, Physical Scripta Vol. 51, (1995), pp. 498{516.
- [13] J.C. New: \Wall-Mediated Forces between Like-Charge Bodies in an Electrolyte\, Phys. Raw. E., Vol. 82(5), (1999), pp. 1072{1074.
- [14] M. Tokuyama: \Elective forces between highly charged colloidal suspensions\, Phys. Rev. E., Vol. 59(3), (1999), pp. R2550{R2553.
- [15] A.N. Tkachev and S.I. Yakovenko: \Electron clouds of charged macroparticles\, Tech. Phys., Vol. 44, (1999), pp. 48{52
- [16] N.N. Stanovich: \Dust plasma crystals, drops and clouds\, USPHS Finishes Neuk, Vol. 167(1), (1997), pp. 57{99 (in Russian).
- [17] S.I. Yakovenko: \About Interaction of Charged Particles. Debye Molecule\, Technical Physics Letters, Vol. 25, (1999), pp. 670{672.
- [18] S.I. Yakovenko: \Plane thermionic clouds and charge of dust particles\, Technical Physics Letters, Vol. 26, (2000), pp. 337{341.
- [19] S.I. Yakovenko: \Ion Recombination on Dust Particles in a Dense Gas Plasma Excited by a Hard Ionizing Factor\, Technical Physics Letters, Vol. 26, (2000), pp. 1045{1048.
- [20] S.I. Yakovenko: \The Interaction of Charged Planes in an Electron Cloud and Plasma\, Technical Physics Letters, Vol. 27(5), (2001), pp. 389{393.